467

VOLUME 80 SEPARATE No. 467

PROCEEDINGS

AMERICAN SOCIETY
OF
CIVIL ENGINEERS

AUGUST, 1954

TRANQUIL FLOW THROUGH OPEN-CHANNEL CONSTRICTIONS

by Carl E. Kindsvater, M. ASCE, and
Rolland W. Carter, A.M. ASCE

HYDRAULICS DIVISION

{Discussion open until December 1, 1954}

Copyright 1954 by the AMERICAN SOCIETY OF CIVIL ENGINEERS
Printed in the United States of America

Headquarters of the Society
33 W. 39th St.
New York 18, N. Y.

PRICE $0.50 PER COPY

THIS PAPER

--represents an effort by the Society to deliver technical data direct from the author to the reader with the greatest possible speed.

Readers are invited to submit discussion applying to current papers. For this paper the final date on which a discussion should reach the Manager of Technical Publications appears on the front cover.

Those who are planning papers or discussions for "Proceedings" will expedite Division and Committee action measurably by first studying "Publication Procedure for Technical Papers" (Proceedings — Separate No. 290). For free copies of this Separate—describing style, content, and format—address the Manager, Technical Publications, ASCE.

Reprints from this publication may be made on condition that the full title of paper, name of author, page reference (or paper number), and date of publication by the Society are given.

The Society is not responsible for any statement made or opinion expressed in its publications.

This paper was published at 1745 S. State Street, Ann Arbor, Mich., by the American Society of Civil Engineers. Editorial and General Offices are at 33 West Thirty-ninth Street, New York 18, N. Y.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

TRANQUIL FLOW THROUGH OPEN-CHANNEL CONSTRICTIONS

By Carl E. Kindsvater,[1] M. ASCE and Rolland W. Carter,[2] A. M. ASCE

[1] Prof. of Civ. Eng., Georgia Inst. of Technology, Atlanta, Ga.

[2] Staff Engr., Geological Survey, U. S. Dept. of the Interior, Surface Water Branch, Water Resources Div., Atlanta, Ga.

Synopsis

Area constrictions in many forms are encountered in both natural and artificial open channels. The hydraulics of transitions, control sections, and bridge piers and piles has been extensively investigated in the field and in the laboratory. It is remarkable, therefore, that most of the investigations of obstructions in bridge waterways have ignored the fact that the waterway itself is a definitive example of a width constriction in the river channel. The bridge waterway is studied in the basic research described in this paper.

The effect of an area constriction on the hydraulics of steady, open-channel flow depends on the geometry of the flow boundaries, the rate of flow, and the physical properties of the fluid. The resulting flow pattern is not subject to a general analytical solution. A practicable solution of the discharge equation has been achieved by the application of a systematic experimental investigation to an approximate analysis. Empirical data, which extend the solution to a wide variety of boundary conditions, are presented in dimensionless, graphical form. A significant feature is a detailed description of the external and internal characteristics of the flow pattern.

Notation

The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically for convenience of reference in the Appendix.

Introduction

Purpose of the Investigation.—For several years the Geological Survey, United States Department of the Interior (USGS), has conducted field and office investigations of various "indirect" methods for determining flood discharge. Among these methods was the "contracted-opening" method. Certain unexplained inconsistencies, revealed in these investigations, resulted in the authorization of a program of fundamental research on the contracted-opening method. The purposes of the research, begun in February 1951, were (1) to acquire a critical knowledge of the mechanics of the flow of water through width constrictions in open channels, and (2) to develop, if possible, a practicable method (based on field surveys of flood marks and channel characteristics) for computing the discharge through bridge waterways.

It was immediately recognized that the attainment of the second objective would depend entirely on a preliminary success in accomplishing the first objective. It was also realized that the flow pattern involved was not subject to a general analysis and that the attainment of the first objective would depend largely on a carefully controlled experimental investigation of certain basic boundary forms. It was anticipated that the laboratory study would not only establish the relative influence of the various flow parameters and primary geometric variables, but that a general method of evaluating the discharge function could be developed and subsequently verified by a systematic examination of the influence of additional boundary conditions. Thus, during the first phase of the investigation, the ultimate objective was subordinated to an intensive investigation of the arbitrarily defined fundamental problem of tranquil flow through open-channel constrictions. This part of the research is believed to have general significance as a basic tool in applied fluid mechanics.

Scope of the Investigation.—The flow of water through bridge waterways was the definitive problem in the determination of the scope of the research. Normally, this example of rapidly varied motion involves steady, turbulent flow, entirely in the tranquil range. The submerged parts of the highway embankments, bridge abutments, and piers comprise an abrupt width constriction in the natural channel.

The variety of natural boundary conditions, considering the adjacent channel reaches and the constrictive elements, is almost unlimited. It was therefore necessary to restrict the basic investigation to certain elementary boundary forms which could be easily described and conveniently reproduced in the laboratory. Thus, the constrictions used in the experimental studies were limited to the forms made possible by the use of vertical walls of various lengths, widths, and corner roundings. Limitations on the relative size and location of the constrictions, the range of stream velocities, depths of flow, channel configurations, and channel roughnesses were governed by an arbitrary definition of the normal, single-opening bridge waterway.

The effect of piers and piles, multiple openings, and sloped embankments were not investigated during this phase of the research. By restricting the study to tranquil flows through constrictions simulating bridge waterways, certain problems which are associated with shaped transitions in artificial channels were also excluded. These exclusions were justified, in part, by the

fact that other investigations[3,4,5,6,7] had provided satisfactory solutions to

[3] "The Hydraulic Design of Flume and Siphon Transitions," by J. Hinds, *Transactions*, ASCE, Vol. 92, 1928, p. 1423.

[4] "The Flow of Water in Flumes," by F. C. Scobey, *Technical Bulletin 393*, U.S.D.A., Washington, D. C., December, 1933.

[5] "Experiments on the Flow of Water Through Contractions in an Open Channel," by E. W. Lane, *Transactions*, ASCE, Vol. 83, 1919–1920, p. 1149.

[6] "High-Velocity Flow in Open Channels: A Symposium," *ibid.*, Vol. 116, 1951, p. 265.

[7] "Channel Transitions and Controls," by Arthur T. Ippen, *Engineering Hydraulics*, edited by H. Rouse, John Wiley and Sons, Inc., New York, N. Y., 1950, Chapter VIII, p. 496.

many aspects of the problems related to transitions and critical-flow control sections. It is of interest to note that the variety of boundary conditions considered is equal to, if not greater than, that covered by available information on the orifice, which is the closed-conduit counterpart of the problem investigated.

Review of the Literature.—Information (published prior to 1953) on the theory and methods of computing flow through open-channel contrictions is both rare and inconclusive. Ivan E. Houk,[8] M. ASCE, reporting methods of

[8] "Calculation of Flow in Open Channels," by Ivan E Houk, *Technical Reports*, Miami Conservancy District, Dayton, Ohio, Part IV, 1918.

computing flood discharges used by the Miami (Ohio) Conservancy District, was probably the first to describe the contracted-opening method. Mr. Houk credited S. M. Woodward for the procedure used by the District. Mr. Houk showed that the discharge equation underlying Mr. Woodward's analysis was identical with that which described the flow through a venturi tube—that is, a form of the combined energy and continuity equations. The discharge equation offered by Mr. Houk included a coefficient which was primarily a contraction coefficient.

E. W. Lane,[5] M. ASCE, is the first known investigator in the United States to have studied simple width constrictions in the laboratory. However, his work was concerned primarily with higher Froude numbers than are usually involved in natural flow under bridges, and the variety of boundary forms studied by Mr. Lane was extremely limited.

D. L. Yarnell, in his research on flow past bridge piers, made an extensive study of the literature, which is annotated in his report.[9] Mr. Yarnell's ex-

[9] "Bridge Piers as Channel Obstructions," by David L. Yarnell, *Technical Bulletin 442*, U.S.D.A., Washington, D. C., November, 1934.

periments ignored the effect of the bridge abutments. For his analysis, Mr. Yarnell selected

> "the bridge-pier formulas most commonly used in the United States * * * d'Aubuisson's, Nagler's, Weisbach's, and Rehbock's."[9]

Mr. Yarnell dismissed as unsound the Weisbach formula, which assumes that the total flow through a constricted section is the sum of an orifice discharge and a weir discharge. This conclusion, which Mr. Houk had previously stated, is generally accepted. The d'Aubuisson formula, in its basic form, is simply the discharge equation which results from a simultaneous solution of the continuity and energy equations. Therefore, the resulting equation is essentially the same as that suggested by Mr. Woodward, and should not be termed an empirical "formula." The Nagler and Rehbock equations are essentially forms of the discharge equation which have been adapted specifically

to the bridge-pier problem. These equations are more logically termed empirical formulas.

In 1944, the USGS began using a procedure for estimating the flow through area-constrictions which was based on a form of the d'Aubuisson equation. The USGS (as a result of Mr. Houk's work and of a limited number of field observations) used values of the coefficient of discharge that were related to the degree of rounding of the upstream edges of the constriction.

Laboratory Arrangement.—Laboratory tests made as a part of the investigation were conducted in the hydraulics laboratory of the Georgia Institute of Technology, at Atlanta, through arrangements with the Georgia Tech Research Institute. The principal item of equipment was a steel flume 18 in. high and 10 ft wide, with a usable, uniform-flow length of approximately 21 ft. Water was supplied from the laboratory's recirculating system. Discharges were measured by use of either a venturi meter or a bend meter, both located in the supply piping. Water levels were measured by manometer-connected floor piezometers, portable static tubes, and point gages. Velocities were determined by a small (Pygmy) current meter and pitot tubes.

Constrictive elements and false walls were constructed of concrete, aluminum, or exterior-grade plywood. Artificial channel roughness was created by various arrangements of chain-link fencing fastened to the smooth, concrete floor of the flume. Experimentally determined values of Manning's n corresponding to the three degrees of roughness investigated were 0.012 (for the smooth floor), 0.027, and 0.036. Irregular channel cross sections were produced by placing various arrangements of $\frac{1}{2}$ in.-thick, cement-asbestos board on the floor of the flume. Manning's n for the board (smooth side) was determined to be 0.011 within the range of conditions tested.

Description of the Flow Pattern

General.—There is particular reason for including a detailed examination of the external and internal flow characteristics in an investigation of the open-channel constriction.

Since the experimental investigation must be limited to certain basic boundary conditions, the successful application of the results to practical problems will depend, to a large extent, on a critical knowledge of the internal mechanism of the flow. Equally important, however, is an understanding of the external characteristics of the flow pattern, so that the data required for proper application of the results can be determined. There is ample evidence that the failure to recognize the critical variables has accounted for many of the inconsistencies attributed to the contracted-opening method of computing discharge.

The External Picture.—Fig. 1 illustrates certain basic boundary conditions and hydraulic characteristics of flow through simple width constrictions in rectangular open channels. The beginning of acceleration of the central body of water approaching an open-channel constriction is indicated by a slight decrease in water surface level at a distance upstream from the face of the constriction approximately equal to the width of the opening. Deceleration occurs along the outer boundaries, and a separation zone is created in the

corner adjacent to the constriction (zone 1a in Fig. 1). The size of the separation zone and the amount by which its surface level differs from the full stagnation level is a function of the upstream channel characteristics and the geometry of the constriction.

Water from the sides enters the opening as a sharply curved and contracting stream, indicating that it is being accelerated in directions both normal and parallel to the streamlines. Just as the longitudinal acceleration and the consequent negative pressure gradient is reflected as a drop in the longitudinal profile, the normal acceleration accounts for a considerable difference in level between the outside filaments of flow and the central filaments of flow. It generally happens that the greater part of the total drop in average surface level occurs very near the upper face of the constriction.

As the water passes through the constriction, the contracted stream reaches a minimum width at section 2 which corresponds to the vena contracta in orifice flow. The flow is bounded here by an eddying body of water in zone 3 which marks a second separation zone. Expansion of the live stream begins at the vena contracta and ends at the section of normal regimen in the full-width channel section 4. The length of channel required for this process depends on the boundary configuration, the roughness of the channel, and the magnitude of the pertinent flow parameters. Hunter Rouse,[10] M. ASCE, working with a two-dimensional orifice discharging into the atmosphere, found that the nominal boundary of the expanding jet would have a slope of 1 on 4 with respect to the longitudinal axis. H. R. Henry[7,10] working with the two-dimensional discharge below a sluice gate (wherein the expansion is limited by a fixed lower boundary and a free-surface upper boundary) reported a slope of approximately 1 on 6.

[10] "Diffusion of Submerged Jets," by M. L. Albertson, Y. B. Dai, R. A. Jensen, and Hunter Rouse, *Transactions*, ASCE, Vol. 115, 1950, p. 639.

The average level of the stream at the vena contracta (section 2) is approximately equal to the level in the downstream separation zone (3). The level of the fully expanded jet at section 4 is usually higher than that in zone 3. The latter relationship is a function of the shape and friction characteristics of the channel between these sections.

For constrictions of considerable length (measured parallel to the flow), eddying water fills the space (3b) between the face of the constriction and the contracted stream. Under these circumstances, the level of the water in this space is frequently a good measure of the average level of the surface at the vena contracta.

It is apparent, therefore, that certain factors which have a dominant influence on the flow pattern deserve closer attention. It is of particular importance to examine the effect of the fixed boundaries, the separation boundaries, and the upper, free-surface boundary on the distribution of velocities and surface elevations over the flow pattern. Under similar circumstances, a flow net is frequently utilized. In this case, however, the same restrictions which prohibit the accomplishment of a general analytic solution also prohibit the construction of a flow net. By temporarily ignoring the influence of the free surface, however, an approximate flow net can be drawn on the basis of an analogy with the classic, two-dimensional orifice.

The Orifice Analogy.—Within the specified limits of the investigation, the accelerated motion in the vicinity of an open-channel constriction can be considered to be primarily two-dimensional in the horizontal plane. Subject to obvious theoretical and practical limitations, it follows that the flow pattern can be approximated by a flow net drawn for a two-dimensional constriction.

Fig. 2(*b*) shows a thin-plate orifice, or slot, located symmetrically in—and normal to—parallel boundaries. The fluid flowing is assumed to be incompressible, and it is assumed to fill the space downstream from the orifice. The flow net was constructed by approximate, graphical methods. In its preparation, it was assumed that velocities in the approach section were uniformly distributed between the boundaries, but that separation would occur in the corner upstream from the constriction. These are contradictory assumptions, because the first implies irrotationality (corequisite to the assumption of a non-viscous fluid), and the second implies rotationality. Furthermore, the separation boundary shown in Fig. 2(*b*) is an arbitrarily assumed line, the exact location of which is indeterminate. These contradictions, which were required for the purposes of the analogy, are not unusual. The resulting flow net is not expected to be seriously in error for the case of smooth boundaries and high values of the Reynolds number.

It should be emphasized that the purpose of introducing the orifice analogy is to facilitate a qualitative—rather than a quantitative—analysis of the phenomenon under investigation. In fact, the main purpose is to make Fig. 2(*b*) available for the subsequent analysis of the internal flow picture. Fair agreement between the three-dimensional, open-channel flow pattern and the analogous two-dimensional pattern are expected only when the former is restricted to tranquil, turbulent flows in smooth, regular channels, with low to moderate degrees of channel contraction. The investigation of conditions outside this range are benefited indirectly by the analogy.

The Internal Picture.—Two specific occurrences of separation are indicated in Figs. 1 and 2(*b*). In the first instance, separation occurs near the corner formed by the conduit boundary and the upstream face of the constriction. The separation at this point is consistent with the admission that the flow of real fluids is accompanied by the existence of a boundary layer. Thus, the normal deceleration tendencies in the corner immediately adjacent to the boundary (where the limiting boundary-layer velocity is already zero) result in the reverse flow which initiates separation. The separation zone, designated as zone 1a in Fig. 2(*b*), is a region of eddying fluid under an essentially constant piezometric head equivalent to the piezometric head along the bounding streamline. Thus, the separation prevents the occurrence of a full stagnation head in the corner. The separation also influences the piezometric head and the velocity distribution over the adjacent flow pattern. The velocity discontinuity at the separation boundary creates eddies which result in a loss of energy.

The second instance of separation, shown in Figs. 1 and 2(*b*), is marked by the boundary between the contracted stream and the eddying fluid in the corners downstream from the constriction. Unlike the upstream separation zone, this zone is fixed by the boundary conditions and the assumption of

irrotationality of the live stream. Thus, the jet outline is assumed to be identical with the free-surface boundary which would exist if the stream were not enclosed by the fluid flowing. Here, also, the piezometric head in the separation zone is essentially constant and equal to the piezometric head on the bounding streamline. From the properties of the flow net, furthermore, it is seen that the piezometric head in the separation zone must be equal to the piezometric head across the vena contracta. This conclusion is substantiated by the observation that from Newton's second law, a pressure gradient cannot exist across the boundary of the jet where that boundary is a straight line, for in that instance there is zero acceleration normal to the flow. It also follows from this principle that the pressure in the zone adjacent to the downstream side of the constriction (zone 3a in Fig. 2(*b*)) is essentially equal to that in zone 3 opposite the vena contracta, because of the negligible accelerations which exist between these points.

Expansion of the Contracted Stream.—A description of the internal flow picture is not complete without an explanation of the expansion or diffusion and the resulting deceleration of the contracted stream downstream from the constriction. A knowledge of this process is of little significance in relation to the discharge equation, but is prerequisite to an understanding of the energy transformations which occur downstream from the constriction.

In Fig. 2(*b*) the idealized boundary which divides the downstream separation zone from the live stream is not indicative of the disturbed flow which ordinarily characterizes this region. The boundary actually exists as a fairly smooth, steady surface only when a heavy fluid, such as water, discharges into a large space occupied by a relatively light fluid, such as air. When (as is assumed in this case) the fluid discharges into an enclosed space occupied by a fluid of similar density, the extreme velocity gradient at the separation boundary results in shear stresses sufficient to initiate violent eddy action. The end effect is a condition of instability wherein the kinetic energy of the live stream is steadily converted into turbulent energy, which is eventually dissipated through viscous shear.

As shear along the separation boundary results in deceleration of the live stream, the width of the jet increases in the direction of flow in accordance with the continuity relationship. Eddies created along the boundary result in lateral mixing of the two bodies of fluid until eventually the diffused jet occupies the full area of the cross section. Just as the fluid in the jet is decelerated, the fluid originally in the separation zone below the constriction is accelerated, entrained, and carried downstream by the mixing process. For steady flow to be maintained within the fixed boundaries, therefore, equal quantities of fluid must be exchanged across the boundary of the expanding flow. The circulatory flow which occurs in the separation region is thus a consequence of the mixing process and the slight pressure gradient which exists along the conduit boundary between the constriction and the expanded jet.

Distribution of Piezometric Head.—The assumption of irrotationality within the limiting streamlines shown in Fig. 2(*b*) permits the application of the constant-energy principle to all points in the flow. From the properties of the slow net, therefore, an equation can be written which relates the variation

in piezometric head to the relative spacing of the streamlines. The piezometric head at any point on the surface of an open-channel flow (in which pressure is hydrostatically distributed) is equivalent to the elevation of that point. This assumption is valid if the accelerations in the vertical direction are relatively small. In this case, the only region in which vertical accelerations might account for minor departures from hydrostatic pressure distribution is in the region of maximum total acceleration, near the upstream corner of the constriction. Thus, the orifice flow net, drawn for purposes of analogy, yields an approximate picture of the variation of surface level in the vicinity of a similar open-channel constriction.

Since it is implied (in the assumption of irrotationality) that the fluid is "frictionless," the energy loss between any upstream section of a particular streamtube and a section at the vena contracta can be ignored. Thus, Bernoulli's equation takes the form,

$$\frac{V^2}{2g} + h = \frac{V_2^2}{2g} + h_2 \quad \ldots\ldots (1)$$

in which V is the average velocity in the streamtube cross section; h is the piezometric head, defined as the sum of the pressure head and the elevation; and g is the gravitational acceleration. From the principle of continuity, as applied to the two-dimensional streamtube of width Δn,

$$q = V\,\Delta n = \text{a constant} \quad \ldots\ldots (2)$$

in which q is the unit discharge. From the properties of the flow net, the normal distance between the streamlines, Δn, is equal to the normal distance between the potential lines, Δs, at the same point in the flow. Therefore,

$$\frac{V}{V_2} = \frac{\Delta n_2}{\Delta n} = \frac{\Delta s_2}{\Delta s} \quad \ldots\ldots (3)$$

Combining Eqs. 1 and 3 results in

$$\frac{h - h_2}{\frac{V_2^2}{2g}} = 1 - \left(\frac{\Delta s_2}{\Delta s}\right)^2 \quad \ldots\ldots (4)$$

in which the term to the left of the equal sign is a dimensionless piezometric-head ratio and the term on the right is determined at every point in the flow pattern by the relative spacing of the streamlines. As the geometry of the flow net is fixed by the geometry of the limiting streamlines, the piezometric-head ration is independent of the absolute magnitudes of the reference head, h_2, the reference velocity, V_2, and the size of the orifice. The distribution of the piezometric head along the center line of the conduit and the upstream boundary of the orifice is shown in Fig. 2(*a*). These curves can be assumed to be fair approximations of the variation in the relative surface elevations at corresponding locations at the open-channel constriction. The ordinate scale in Fig. 2(*a*) corresponds to Eq. 4.

Selection of the Independent Variables.—The flow pattern in the vicinity of an open-channel constriction is influenced by the physical properties of the fluid (assumed to be water throughout this paper), the rate of flow, and the shape and roughness of the constriction and adjacent channel reaches. In this case the influence of surface tension and compressibility can be ignored. In the region of comparatively rapid acceleration near the constriction, the influence of viscosity can be assumed to be relatively insignificant. Thus, the influence of fluid weight is presumed to be dominant among the physical forces which govern the motion pattern. The critical flow parameter, therefore, is the Froude number **F** expressed as

$$\mathbf{F} = \frac{V}{\sqrt{g\,y}} \qquad (5)$$

in which y is a significant depth.

Because only tranquil flows have been considered, the Froude number is always less than unity; and because the range of depths is fixed by arbitrarily defined, normal, natural-channel conditions, the velocity and also the unit discharge are limited to comparatively narrow bounds.

The most distinctive influences, therefore, are those which constitute the boundary conditions. Among these influences are many natural boundary characteristics which are not independently expressible in terms of simple mathematical quantities. Others, such as roughness and channel shape, might be incorporated in a uniform-flow equation, but they are conveniently omitted from an analysis of the discharge characteristics of the constriction. For purposes of an approximate analysis, therefore, certain boundary conditions which are peculiar to irregular natural channels are excluded. The subsequent examination considers only rectangular, level channels.

It is required to select the variables which play major, independent roles in influencing the motion pattern previously described. As nothing in the mechanical process of dimensional analysis will indicate the omission of critical independent variables, every possible variable must be examined. Certain significant flow characteristics, such as water levels and velocities, are omitted because they are dependent on variables which are more conveniently incorporated in a practical solution. Certain other variables, which are preferred for describing the analogous orifice pattern shown in Fig. 2(*b*), are necessarily replaced by variables better suited to the description of an open-channel flow.

Dimensional Analysis.—The variables selected and illustrated in Fig. 1 are believed to be sufficient to describe the average flow characteristics. The total discharge is denoted by Q; B is the width of the channel; b denotes the width of the opening; $\Delta h = h_1 - h_2$ is the difference in piezometric heads (or surface levels, assuming hydrostatic pressure distribution) between the approach section (1) and the section of maximum contraction (2). The average depth of flow at section 2 is represented by y_2 (at least one depth must be included, and y_2 is assumed to be best defined as an average); L is the length of

the constriction at the opening, parallel to the flow; and r denotes the radius of the rounding of the upstream corners of the constriction. The density ρ is not included as an independent variable because it is contained in the terms Δh and g.

With the selected variables assembled in a functional relationship,

$$f_1 (Q, B, b, \Delta h, y_2, L, r, g) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (6a)$$

they are combined by methods of dimensional analysis into a minimum number of significant ratios,

$$f_2 \left(\frac{Q}{b\, y_2 \sqrt{g\, \Delta h}}, \quad \frac{Q}{b\, y_2 \sqrt{g\, y_2}}, \quad \frac{b}{B}, \quad \frac{y_2}{b}, \quad \frac{L}{b}, \frac{r}{b} \right) = 0 \quad \ldots\ldots\ldots (6b)$$

The first ratio in Eq. 6*b* is proportional to a coefficient of discharge, C, and the second is equivalent to F for the contracted stream. The other ratios are representative of the principal geometric properties of the boundary.

If Eq. 6*b* is written in the more significant form,

$$C = f_3 \left(\mathsf{F}, \quad \frac{b}{B}, \quad \frac{y_2}{b}, \quad \frac{L}{b}, \quad \frac{r}{b} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (6c)$$

it is seen that, for any given form of boundary, the configuration of the free surface is a unique function of the Froude number of the contracted stream. That is, C is a constant for any given boundary and value of the Froude number.

The Discharge Equation.—The form of the discharge function indicated by Eq. 6*c* is approximated by combining the one-dimensional energy equation and the continuity equation between sections 1 and 2 in Fig. 1. It is appropriate, however, to recognize the two major energy losses which occur between the nominal approach section (1) and the section of maximum contraction (2). The least of these losses, designated as h_e, is the eddy loss caused by the turbulence engendered in the upstream separation zone. The greater loss is h_f, the boundary friction loss, which is primarily a function of the channel characteristics.

An additional consequence of recognizing the influence of the channel configuration and roughness is the introduction of factors that account for the nonuniform velocity distribution in the flow cross section. Thus, letting α represent the ratio of the average kinetic energy to the kinetic energy corresponding to the average velocity, the energy equation takes the form,

$$\alpha_1 \frac{V^2_1}{2g} + h_1 = \alpha_2 \frac{V^2_2}{2g} + h_2 + h_e + h_f \quad \ldots\ldots\ldots\ldots\ldots (7)$$

Introducing C_c, the coefficient of contraction, in the equation of continuity results in

$$Q = B\, y_1\, V_1 = C_c\, b\, y_2\, V_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Solving Eq. 7 and 8 simultaneously for the discharge,

$$Q = \frac{C_c\, b\, y_2}{\sqrt{\alpha_2}} \sqrt{2\,g\,(h_1 - h_2 - h_e - h_f + \alpha_1 \frac{V^2_1}{2\,g}} \quad \ldots\ldots\ldots\ldots (9)$$

The terms h_f and $\alpha_1 \frac{V^2_1}{2\,g}$ are dependent primarily on the resistance characteristics of the approach channel. These terms are relatively independent of the variables which describe the accelerated motion through the constriction. In any practicable computation procedure, therefore, the factors are best evaluated separately for substitution in an equation such as Eq. 9. The terms h_e and α_2, however, are much less dependent on channel shape and roughness than on the geometry of the constriction. Thus, these factors are functions of the variables contained in Eq. 6. Therefore it follows that Eq. 9 can be written as

$$Q = C\, b\, y_2 \sqrt{2\,g \left(h_1 - h_2 - h_f + \alpha_1 \frac{V^2_1}{2\,g} \right)} \quad \ldots\ldots\ldots\ldots (10)$$

If it is assumed that

$$h_e = K_L \frac{V^2_2}{2\,g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

in which K_L is an energy-loss coeffieient. From Eqs. 6c and 10, wherein C is a coefficient of discharge in the form indicated by Eq. 6c,

$$C = \frac{C_c}{\sqrt{K_L + \alpha_2}} = f_4 \left(\mathrm{F},\ \frac{b}{B},\ \frac{y_2}{b},\ \frac{L}{b},\ \frac{r}{b} \right) \quad \ldots\ldots\ldots\ldots (12)$$

In the foregoing description of the flow pattern, it was observed that the elevation of the water surface at the downstream side of the constriction (zone 3*a* in Fig. 1) is essentially equivalent to the elevation of the water surface at the section of maximum contraction (2). Under these conditions, it is possible to compute y_2 as the difference between the average level at 3*a* and the average bottom elevation at 2. In terms of natural channel conditions, therefore, y_2 is approximately the difference between the average level of the water surface at the downstream face of the constriction and the average bottom elevation at a corresponding section across the opening. This is not only a very practicable procedure, but for all conditions (except, perhaps, extreme values of L/b) the computed depth, y_3, is a close approximation for y_2. Eq. 10 thus takes the form,

$$Q = C\, b\, y_3 \sqrt{2\,g \left(\Delta h - h_f + \alpha_1 \frac{V^2_1}{2\,g} \right)} \quad \ldots\ldots\ldots\ldots (13)$$

in which $\Delta h = (h_1 - h_3)$, the difference in water-surface elevations between the nominal approach section (1) and the downstream side of the constriction (3*a*). The discharge coefficient remains a function of the parameters contained in Eq. 12.

Experimental Verification of the Analysis.—In the preceding development the form of the discharge equation was determined by an approximate energy analysis. The significant parameters that govern the coefficient of discharge were determined from dimensional analysis of the assumed variables. The primary purpose of preliminary laboratory tests which were made in connection with this investigation was to substantiate the form of the discharge function and to establish the relative importance of each of the parameters. The functional relationship for C, expressed by Eq. 12, was assumed to be correct only to the extent that it was a dimensionally correct combination of the selected variables. It was assumed that failure to include all of the pertinent variables would be indicated by the inability to correlate all the experimental data by use of the derived parameters.

The preliminary exploratory tests were made in a laboratory reproduction of the basic boundary conditions shown in Fig. 1. These tests not only substantiated the selection of variables but demonstrated that the test procedure could be simplified by ignoring the influence of certain variables for all or part of the practical range. Thus, of the ratios contained in Eq. 12, it was indicated that the width ratio, b/B, is of major importance, while **F**, L/b, and r/b are of lesser but appreciable importance in defining the C-function. The ratio y_2/b, a shape factor, appeared to have a negligible influence for the entire range of conditions which were considered to be practical.

With the ratio y_2/b eliminated, the functional relationship for C in Eq. 12 contains four ratios. Not more than two of these ratios can be shown conveniently, together with C, in any single graphical illustration of experimental results. Because of the necessity of developing a practicable computational procedure, the experimental data from the initial tests were examined in detail. After much study, a procedure was adopted which involved an approximation method for determining the complex interrelationships of the principal variables. The method was then substantiated by demonstrating that it would yield results of sufficient accuracy for the entire practical range of all the variables.

An Approximate Solution for Simple Constrictions.—The procedure adopted involved a base curve of the relationship between C, b/B, and L/b which was first determined for certain arbitrary constant values of **F** and r/b. To complement the base curve, average "adjustment" curves were developed from systematic investigations of the separate influences of the latter ratios. In every case, the adequacy of the average curves was checked throughout a reasonable range of all the independent ratios. With these curves derived from the experiments, a straightforward procedure was achieved for computing the essential characteristics of flow through simple constrictions in level, rectangular channels

Simple Constrictions in Irregular Channels

The General Problem.—An analysis of discharge characteristics and a satisfactory computational procedure have been presented for a definitive, simplified example of an open-channel constriction. For this basic case, the coefficient of discharge was shown to be a function of five independent

ratios, one of which was eliminated as a result of a laboratory investigation. One of the remaining ratios is a flow parameter—characteristic of all gravity-controlled phenomena—the Froude number. The other ratios are descriptive of the geometry of the constriction. The geometry of the channel in which the constriction is located is involved, to only a minor extent, in the evaluation of the friction-loss term in the discharge equation.

However, it is necessary to extend the analysis to the more complex boundary conditions involved in the definitive practical problem—the bridge waterway. Outstanding among these complexities are the configuration and roughness of the channel. It is assumed that the friction loss can be computed by use of the Manning formula or a similar open-channel-flow formula. But the influence of a rough, irregular channel on the efflux characteristics of the constriction is a matter which cannot be evaluated by analytic means, nor can it be approximated by analogy with the common orifice. This, then, is the principal objective of an extended analysis.

Additional Variables in the Extended Analysis.—As specified under the heading "Introduction. Scope of the Investigation," this study is restricted to certain basic boundary conditions. The simplest of these, as illustrated in Fig. 1, were involved in the successful accomplishment of an approximate solution. The extended analysis involves irregular channels of varied roughness patterns, and three additional conditions relative to the shape and arrangement of the constriction in the channel.

The channel in the vicinity of natural-channel constrictions is irregular in the sense that it is comprised of a main channel and one or two overbank, flood channels. The cross-sectional shapes tested in the laboratory are shown in Fig. 3 and the corresponding dimensions are listed in Table 1. As indicated in Fig. 3 and Table 1, the relative width occupied by the overbank channel was assumed to vary, and the constricted waterway was assumed to include various proportions of main channel and flood channel. It was also assumed that each of the component sections might possess different roughness characteristics. Thus, the roughness patterns tested in the laboratory were composed of various combinations of artificial bottom roughnesses, varying, in terms of Manning's n, from 0.011 to 0.036. It was assumed that the channel would have the same shape and roughness characteristics downstream from the constrictions as upstream. Exploratory tests indicated that major variations in the shape of the downstream channel have no significant effect on the discharge characteristics of the constriction.

Anticipating that the channel shape and roughness characteristics do not have to be included in the array of independent variables involved specifically in the discharge function, only three new boundary parameters need be added to those contained in Eq. 12. These parameters are described as eccentricity, angularity, and entrance guide walls.

Whereas it was previously assumed that the constrictions would be symmetrically located with respect to the longitudinal axis of the channel, in the extended analysis it will be assumed that they might be eccentrically located. The degree of eccentricity, represented by the ratio e, is defined as the ratio of the length of the shorter obstruction to the length of the longer obstruction.

In the intitial analysis, it was assumed that the parallel faces of the constrictive element were perpendicular to the parallel boundaries of the channel.

TABLE. 1.—VALUES OF THE VARIABLES SHOWN IN FIG. 3.

Figure	B, in feet	b, in feet	$e = \frac{x_a}{x_b}$	y, in feet
3(a)	2.50 to 10.00	1.00 to 8.00	0.00 to 1.00	
3(b)	5.56 to 9.05	1.34 to 5.23	0.79 to 1.00	0.17 to 0.33
3(c)	9.50	2.40	0.73	0.17

Natural channel boundaries are not necessarily parallel, and the thread of the stream is not necessarily perpendicular to the parallel plane of the constriction. Thus, in the extended analysis, an angle of skew or angularity, ϕ, is defined as the acute angle between the plane of the constriction and a line perpendicular to the thread of the stream.

The third added variable involves the shape of the upstream corners of the constriction. Exploratory tests indicated that rounding the downstream corners has an insignificant effect on the coefficient of discharge. In the initial analysis the upstream corners were taken to be either sharp or rounded to circular arcs. In this analysis it was assumed that the corners might have diagonal, plane faces, simulating vertical guide walls. A single variety of guide walls (set at 45° as indicated in Fig. 7(*b*)), to be shown subsequently was investigated. Verification of the approximate solution as applied to this design should be adequate evidence of the validity of the solution. The ratio W/b is a measure of the relative length of the guide wall.

The additional boundary conditions impose certain qualifications on the parameters included in Eq. 12. Thus, areas cannot be computed as simple products of depths and widths, velocities cannot be assumed to be uniformly distributed across the channel, and depths cannot be referred to a horizontal bottom datum. The most disturbing consequence, however, is the changed significance of the width ratio, b/B.

The Channel-Contraction Ratio.—The width ratio, b/B, was previously used as a convenient measure of boundary geometry. For rectangular cross sections this ratio is proportional to an area ratio. With a uniform velocity distribution across the section, the ratio is also equivalent to a discharge ratio. If the ratio q/Q is substituted for b/B, however, q/Q is seen to possess a greater significance. As illustrated in Fig. 1, q is that part of the total discharge Q which occupies an area of width b in the total cross section upstream from the constriction. To extend this definition to the general case, q is defined as the normal discharge capacity of a channel having the area characteristics of the opening and the roughness characteristics of the approach channel. As ordinarily defined, the "contraction ratio" is a measure of the relative constriction imposed on a given channel. Therefore, if m is defined as the channel-contraction ratio, then

$$m = 1 - \frac{b}{B} = 1 - \frac{q}{Q} = \frac{Q - q}{Q} \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

A significant physical interpretation of the channel-contraction ratio is suggested by the last term in Eq. 14. That is, m can be interpreted as measure of that part of the total flow which enters from the sides into the contracted stream.

The particular advantage of this concept in connection with the general bridge-waterway problem is that the channel-contraction ratio for irregular, natural channels can be computed as a ratio of hydraulic conveyances. Thus, even for the most extreme combinations of varied depths, alinements, and roughnesses in the approach channel, m can be evaluated from

$$m = 1 - \frac{q}{Q} = 1 - \frac{K_q}{K_Q} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

in which K_q is the conveyance of that part of the approach channel occupied by the discharge q, and K_Q is the conveyance of the total section. Conveyance is defined, in terms of Manning's formula, as

$$K = \frac{1.49}{n} A R^{\frac{2}{3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

in which A is the area, and R is the hydraulic radius of the cross sections under consideration.

The General Discharge Equation.—Substituting the three new boundary parameters in Eq. 12, the functional relationship for the coefficient of discharge becomes

$$C = f_5\left(\mathrm{F},\ m,\ \frac{y_3}{b},\ \frac{L}{b},\ \frac{r}{b},\ e,\ \phi,\ \frac{W}{b}\right) \quad \ldots\ldots\ldots\ldots (17)$$

The discharge equation (Eq. 13) is only slightly changed to

$$Q = C\, b\, y_3 \sqrt{2g\left(\Delta h - h_f + \alpha_1 \frac{V_1^2}{2g}\right)} \quad \ldots\ldots\ldots\ldots\ldots (18)$$

To account for irregular cross sections at the constriction, the depth, y_3, is defined as the difference between the water-surface elevation at the downstream side of the constriction, adjacent to the contracted stream, and the average bottom elevation across the opening. It should be emphasized that y_3 is assumed to be an adequate substitute for y_2, the depth of the contracted stream at the vena contracta. The difference between these two depths is small for all but the larger values of L/b, and even in the latter instance, the difference is a function of the variables contained in Eq. 17. Thus, the experimentally determined values of the coefficient of discharge always contain the effect of this approximation.

Verification of the Approximate General Solution

Experimental Procedure.—The verification procedure used for the general solution involved a systematic investigation of the influence of each of the

independent variables contained in Eq. 17. Ideally, this procedure would require that the variation of C with each independent variable be measured for a full range of each of the remaining variables. An almost unlimited variety of conditions would be involved. Actually, considering the minor influence of certain variables, the procedure was simplified by analyzing and correlating the test results as the experiments were made. The only conditions tested, therefore, were those required to define a series of curves capable of correlating the significant variables for a practical range of each. This leads to the procedure described previously for the approximate solution of the simple constriction in rectangular channels. In this case, however, many more tests were needed to develop the average adjustment curves required for the approximate computation procedure. The range of variables tested and the process of developing the empirical relationships are described subsequently.

Computation of the Friction Head.—It was considered sufficient to investigate only three various channel roughnesses in the laboratory. The smoothest boundary condition resulted when a cement-asbestos board (smooth surface up) was used to create irregular channel cross sections in the flume. Manning's n for this surface, as determined by experiment, was 0.011. Manning's n for the concrete floor of the flume was 0.012, which is not sufficiently different from that for the cement-asbestos board to be considered a distinct degree of roughness. With one layer of 11-gage, 2-in. mesh, chain-link fencing covering the floor, the measured n was equal to 0.027. Maximum roughness was created by use of two layers of chain-link fencing, for which n was equal to 0.036. The net flow area used in computing n for conditions involving the wire fencing was the gross area of the section minus the average cross-sectional area occupied by the wire. The average depth was computed as the ratio of the net area to the top width of the section.

It is recognized that the principal effect of increased channel roughness is to increase the differential head across the constriction. Thus, as roughness increases, evaluation of the net velocity head at section 2,

$$\frac{V^2_2}{2g} = \left(\Delta h - h_f + \alpha_1 \frac{V^2_1}{2g} \right) \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

is increasingly dependent on an accurate evaluation of h_f, the loss of head caused by friction between sections 1 and 2. For smooth channels and relatively large values of the channel-contraction ratio, h_f is insignificant relative to the drop in water surface, Δh. For extremely rough channels, espeically those that contain considerable amounts of tree growth and brush, the computation of friction losses in the accelerated flow approaching the constriction is subject to errors of judgment as well as errors in method.

The procedure used in computing h_f in the verification tests was the standard method used by the USGS. Briefly, the method consists first of computing the average energy gradients at the nominal approach section (1) and at the constricted section (2). The energy gradient (slope) is computed as the square of the ratio of the discharge to the conveyance of the section under consideration. The total friction loss is then computed as the head loss in the reach between section 1 and the upstream face of the constriction, plus the head loss

through the constriction (appreciable only for larger values of L). For the upstream reach, the head loss is defined as the geometric mean of the average gradients at the end sections times the length of the reach. The loss in the constriction is the product of the average gradient at the constricted section and the length of the constriction. Acknowledged weaknesses in this method of computation apparently did not affect its validity for the conditions tested.

Location of the Approach Section.—The approach section is defined as the section which marks the beginning of acceleration of the flow approaching the constriction. Since this is not distinctly defined as a line across the flow pattern, it is difficult to determine by measurement in the field as well as it is in the laboratory. For practical purposes in this case, the beginning of acceleration (or beginning of drawdown as it is frequently termed) is of most concern at the boundaries. In terms of high-water marks, it is desirable to be able to determine the beginning of the drawdown on the upstream face of the embankment as well as on the banks of the stream. The error which results from inaccurate determination of this location is reflected, in the discharge equation, as an error in Δh. When the friction head is small, therefore, this error can be avoided by making a generous estimate of the distance from the constriction to the approach section. When the friction head is appreciable, the assumption of too long an approach section results in a small positive error in the computed value of h_f. This is seldom of significance in the range of conditions considered.

The location of the approach section is influenced primarily by the Froude number and the channel-contraction ratio. Tests were made mainly for the purpose of establishing a rough general rule for locating the nominal approach section. The tests indicate that a convenient and adequate approximation of the length of the approach reach, measured along the center of the channel, is a distance equal to the gross width of the opening, b.

The field conditions for which high-water marks along the upstream face of the embankment may ordinarily comprise the best source of data are those corresponding to larger channel-contraction ratios. In this case, for values of m greater than 0.80, the average water-surface elevation in the approach section may be approximated as the elevation of the water at a point a distance equal to one half the width ($b/2$) from the nearest corner of the opening.

Computation of the Average Kinetic Energy.—An additional effect of the channel shape and the boundary roughness is reflected in the magnitude of the factor α_1 in the kinetic energy term $\alpha_1 \frac{V^2_1}{2g}$. This term and the term h_f in the discharge equation, Eq. 17, are generally of approximately equal magnitude for the conditions considered. Both quantities are ordinarily small in comparison with the total measured drop, Δh. The coefficient α_1 was evaluated on the basis of detailed velocity surveys for several conditions of flow. It was found to vary from 1.06, for relatively large depths of flow in irregular sections, to 1.10 for smaller depths in rectangular channels. These average values were used in the computations. The error involved in estimating α_1 for a natural channel is ordinarily smaller than the error involved in estimating n.

Evaluation of the Discharge Coefficient.—The approximate computation procedure described in connection with the verification of the basic analysis will

also be used for the expanded analysis. Previously, this procedure necessitated the determination of a "standard" empirical relationship between the three most critical independent variables contained in the functional equation for C, Eq. 14. The variables selected for this purpose are C, m, and L/b. Ignoring the ratio y_3/b, which was shown by experiment to be insignificant, the standard condition is then defined as that for which the remaining ratios are constant and equal to the following values: $\mathbf{F} = 0.5$; $r/b = 0$, $e = 1$, $\phi = 0°$, and $W/b = 0$. From the experimental data for the standard condition, a family of base curves showing the relationship between C, m, and L/b was constructed. Individual "adjustment" curves were then developed from systematic investigations of the separate influence of the ratios $\mathbf{F}$, r/b, e, ϕ, and W/b. In each case, as for the basic analysis, the adequacy of the average adjustment curves was then tested throughout a reasonable range of all the related ratios.

The method of applying the experimentally determined functional relationships to the computation of the coefficient of discharge is as follows: For a given case, C' is equal to the "standard" value of the coefficient of discharge, corresponding to the standard values of $\mathbf{F}$, r/b, e, ϕ, and W/b, and the given values of m and L/b; k_F denotes a coefficient which, multiplied by C', will yield a coefficient of discharge adjusted for the effect of the given Froude number, $\mathbf{F}$; k_r is a coefficient similar to k_F which adjusts the coefficient of discharge for the given corner-rounding ratio, r/b; k_e represents a coefficient similar to k_F which adjusts the coefficient of discharge for the given eccentricity, e; k_ϕ is a coefficient similar to k_F which adjusts the coefficient of discharge for the given angularity, ϕ; and k_w denotes a coefficient similar to k_F which adjusts the coefficient of discharge for the given guike-wall ratio, W/b (this coefficient cannot exist concurrently with $r/b > 0$). It follows that the adjusted coefficient of discharge can be computed from the equation,

$$C = (k_F\, k_r\, k_e\, k_\phi\, k_w)\, C' \leqq 1.0 \qquad (20)$$

Effect of the Channel-Contraction Ratio.—Experimental verification of the concept of the channel-contraction ratio in terms of conveyances, Eq. 15, is one of the most significant results of this investigation. As applied to channel sections of all shapes and roughness patterns, m is a measure of the ratio of that part of the flow which approaches the opening from bank areas upstream from the constriction to the total flow which passes through the constriction. The coefficient of discharge, C, was previously described as being primarily a coefficient of contraction. It is logical, therefore, that with all other independent variables constant, increasing values of m should result in decreasing values of C. That is, the degree of "jet" contraction should be proportional to the ratio m. Substantiation of this concept establishes m as a general boundary-geometry parameter.

The variation of C with m is shown in Fig. 4(*a*) for two values of L/b. It can be seen that C varies from a maximum value of 1.0 to a minimum of less than 0.70. These values, as expected, are in reasonable agreement with values of C_c for orifices. The range of (L/b)-values for which this relationship was tested varied from 0 to 1.0, and the test points shown in Fig. 4(*a*) are indicative of the scatter of the experimental data.

Fig. 4(*b*), based on a large number of tests similar to those shown in Fig. 4(*a*), shows the base curves for the approximate computation procedure. In this figure, the coefficient of discharge is indicated by the symbol C', the "standard" value, corresponding to specified constant values of the secondary variables, $\mathbf{F}$, r/b, e, ϕ, and W/b. The standard values of the secondary variables for Fig. 4(*b*) are the same as in Fig. 4(*a*).

It is significant that the definition of the curves shown in Fig. 4(*b*) was not only substantiated but was refined by plotting on the original a great many values computed from Eq. 20. Thus, Fig. 4(*b*) represents the composite of experiments made for a wide variety of channel configurations and different, nonstandard values of the secondary variables. The successful correlation of these data constitutes the most critical step in the verification of the approximate solution.

Effect of the Length Ratio.—The variation of C with the length ratio L/b is indicated by the vertical spread of the curves in Fig. 4(*b*). For all of the conditions tested, L was measured as the perpendicular distance from the upstream face to the downstream face of the constriction. The width of the opening was measured as the minimum clear width between the vertical faces of the opening.

The primary effect of length, as indicated in Fig. 4(*b*), is to increase the coefficient of discharge by decreasing the contraction of the stream as it passes through the opening. That is, the contraction is suppressed, even at small values of L/b, by the negative pressure gradient which exists between the live stream and the eddying fluid which surrounds it. The existence of the pressure gradient is the result of the entrainment of the eddying fluid and the reduced opportunity for fluid in the downstream separation zone to replace that which is entrained. Thus, as L/b is increased, the area of access for the return flow (at the downstream corner of the opening) is increasingly constricted by the expansion of the stream, and the surface level in zone 3b in Fig. 1 tends to drop further below that at the end of the opening, at zone 3a. Simultaneously, however, this difference in level is decreased by the energy loss which results from the expansion process. As it is the level at zone 3a which is used in computing y_2 in the discharge equation, the effect of the first influence is to increase the apparent coefficient of discharge, and the effect of the head loss is to decrease the coefficient of discharge. The growth of these conflicting influences is indicated by the relative spacing of the curves in Fig. 4(*b*).

Effect of the Froude Number.—The Froude number can be defined as a ratio of a typical unit inertial reaction to a typical unit weight force involved in a given fluid motion. Thus, a low value of the Froude number at any point in a free-surface flow pattern ordinarily indicates that gravity, or fluid weight, has a large influence on the motion at that point. In this case, the prerequisite of tranquil flow limits the Froude number in the contracted stream to values less than the critical value. Within this low range, however, larger values of $\mathbf{F}$ for the contracted stream are usually indicative of a relatively large drop in water level, Δh, and a relatively large change in Froude number between the approach section and the constricted section for any given boundary. Furthermore, the effect of gravity on the motion is partly removed by

the condition of "submergence" which exists at the downstream side of the opening. Thus, the relative influence of the Froude number is proportional to Δh, which, for any given boundary geometry, increases with increasing values of **F** for the contracted stream.

The effect of the Froude number on the coefficient of discharge is indicated in Fig. 5, the first of the empirical adjustment curves. Here the value of **F** is the Froude number for the contracted stream and is computed from the equation,

$$\mathbf{F} = \frac{Q}{b\, y_3 \sqrt{g\, y_3}} \qquad (21)$$

Typical data plotted in Fig. 5 demonstrate the apparent lack of correlation between $k_{\mathbf{F}}$, the adjustment ratio, and m, the channel-contraction ratio. The modifying influence of the remaining independent variables, as determined by check tests, is similarly insignificant. The increasing positive slope of the adjustment curve with larger values of **F** is an indication of the increasing influence of the Froude number as the flow conditions approach the critical.

Effect of Entrance Rounding.—The most common form of "streamlined" entrance for open-channel constrictions is that in which the upstream corners are rounded to circular arcs. Generally, identical forms are used for both the upstream and downstream corners, but the influence of the downstream streamlining is not well established. In this case, the laboratory tests indicated that the effect of rounding the downstream corners was negligible.

The effect of rounding the upstream corners of the constriction was determined from tests covering a practical range of the relative radius of curvature, r/b. The results of these tests are shown in the form of adjustment curves in Fig. 6(*a*). It was necessary to include a third variable, m, in order to correlate the results with the standard values of the remaining independent variables.

The obvious effect of rounding is to suppress the contraction and increase the coefficient of discharge. The curves in Fig. 7 show, in general, that C increases at a diminishing rate until it reaches a maximum value. This maximum value of C corresponds to a value of C_c approaching unity. Under these limiting conditions, therefore, the coefficient of discharge reflects the boundary friction loss.

Effect of Eccentricity of Opening.—The opening is described as being eccentrically located when the length of one of the obstructions which comprise the constriction is longer than the other. It would appear that the principal effect of eccentricity would be to change the significance of the channel-contraction ratio in the functional relationship for C. The experiments show that the effect of e on the standard value of C is so small, however, that it can be ignored in most cases. The adjustment curve for the effect of eccentricity is shown in Fig. 6(*b*).

Effect of Angularity.—When a constriction is skewed with respect to the main thread of the stream, the effective area of the opening is decreased. It follows that the coefficient of discharge is decreased in proportion to the magnitude of the skew angle, ϕ. In Fig. 7(*a*), experimental results on the effect of ϕ are plotted as an adjustment curve for the standard value of C. The channel-contraction ratio is an important third variable. As indicated

in Fig. 7(*a*), the limiting value of k_ϕ, corresponding to $m = 0$, is equal to the cosine of ϕ. For this condition, of course, the channel is not constricted. It is apparent from Fig. 7(*a*) that the effect of angularity is very small for values of ϕ less than 20°.

Effect of Guide Walls.—The influence of diagonal guide walls at the entrance to the constriction is similar to that of streamlining. Only one form of guide wall was investigated in the laboratory. This involved plane, vertical walls set at an angle of 45° with the face of the constriction. Fig. 7(*b*) shows the effect of the guide walls on the standard value of the coefficient of discharge. The shape of the curves in Fig. 7(*b*) is logically similar to that shown by the rounding-ratio curves (Fig. 6(*a*)).

Comparison of Adjusted and Observed Values of the Discharge Coefficient.—Verification of the approximate solution throughout the complete range of variables considered is shown in Fig. 8. In Fig. 8, C_{adj} is the coefficient of discharge computed from Eq. 20 on the basis of the base coefficient diagram, Fig. 4(*b*), and the adjustment curves, Figs. 5 to 7, inclusive. The "observed" value of the coefficient, C_{obs}, is computed directly from Eq. 17 on the basis of the measured values of the primary variables.

The ratio of C_{adj} to C_{obs} is a measure of the relative accuracy of the solution. This ratio is plotted as a function of C_{obs} in Fig. 8. The indicated accuracy is satisfactory within the limits of the conditions tested. These limits are tabulated in Fig. 8.

Illustrative Example.—The recommended procedure for obtaining the adjusted coefficient of discharge from the empirical data is illustrated in the following example: These measured data correspond to a test condition arranged in a smooth, rectangular laboratory flume: $Q = 0.710$ cu ft per sec, $y_1 = 0.305$ ft, $y_3 = 0.255$ ft, $B = 10.0$ ft, $b = 2.00$ ft, $r = 0.083$ ft, $L = 0.81$ ft, $e = 1.00$, and $\phi = 0°$. Computed data are: $\Delta h = 0.050$ ft, $h_f = 0.001$ ft, $\alpha_1 \frac{V_1^2}{2g} = 0.001$ ft, $m = 0.80$, $\frac{r}{b} = 0.042$, $\frac{L}{b} = 0.41$, and $F = 0.49$. For these conditions, C_{obs} computed from Eq. 17 is 0.774. This example involves only two adjustment ratios, k_F and k_r. Taking these from Figs. 5 and 6(*a*), respectively, for the conditions indicated, $k_F = 0.995$, and $k_r = 1.085$. The corresponding standard value of the discharge coefficient is $C' = 0.705$. Thus, the adjusted value of the coefficient is $C_{adj} = k_F\, k_r\, C' = 0.995 \times 1.085 \times 0.705 = 0.761$. The relative accuracy of the adjustment procedure in this case is indicated by the ratio $\frac{0.761}{0.744} = 0.984$, which is plotted in Fig. 8 ("Example").

Conclusions

A practicable method of solving the discharge equation for the tranquil flow of water through open-channel constrictions is described. The functional relationship between the coefficient of discharge and the principal independent variables is presented in the form of empirical data which were the result of a systematic laboratory investigation. Boundary conditions considered included vertical constrictive elements, channel cross-sectional shapes, and roughness patterns. Within the limits tested, the proposed computation procedure yields satisfactory results.

Applicability of the proposed solution to bridge waterways in natural channels will depend, to large extent, on a critical analysis of the factors controlling the flow pattern. For this reason, a significant part of the paper is devoted to a detailed discussion of the flow pattern.

Acknowledgments

Much of the credit for the conception and accomplishment of this investigation is due Tate Dalrymple, A.M. ASCE, USGS, Washington, D. C., and M. T. Thomson, A.M. ASCE, District Engineer, USGS, Atlanta. Preliminary studies of field data which emphasized the need for the present study were made by R. W. Carter and Jack Terry, A.M. ASCE, USGS, Denver, Colo. Engineer personnel of the Atlanta District, USGS, participated in the laboratory investigation.

APPENDIX. NOTATION

The following letter symbols, adopted for use in the paper and the guidance of discussers, conform essentially with American Standard Letter Symbols for Hydraulics (ASA-Z10.2-1942), prepared by a committee of the American Standards Association with Society representation, and approved by the Association in 1942:

- A = area of channel cross section;
- B = width of the channel;
- b = width of the opening;
- C = coefficient of discharge;
- C' = "standard" value of the coefficient of discharge;
- C_c = coefficient of contraction;
- e = eccentricity ratio;
- F = Froude number;
- g = acceleration caused by gravity;
- h = piezometric head;
  - h_e = eddy loss between sections 1 and 2;
  - h_f = boundary friction loss;
- K = conveyance, from Manning's formula;
- K_L = energy loss coefficient;
- k = adjustment coefficient relating the adjusted to the standard coefficient of discharge;
  - k_e = adjustment coefficient for the effect of eccentricity;
  - k_{F} = adjustment coefficient for the effect of the Froude number;
  - k_r = adjustment coefficient for the effect of the corner-rounding ratio;
  - k_w = adjustment coefficient for the effect of the guide-wall ratio;
  - k_ϕ = adjustment coefficient for the effect of angularity;
- L = length of the constriction;
- m = channel-contraction ratio;

n = distance measured normal to the streamlines or the retardation coefficient in the Manning formula;
p = pressure intensity;
Q = total discharge at a channel cross section;
q = unit discharge in a rectangular section, or a part of the total discharge through a constriction;
R = hydraulic radius;
r = radius of a circular arc;
S = hydraulic slope;
s = distance measured parallel to streamlines;
V = average velocity;
v = velocity at a point;
W = width of the entrance guide walls;
x = linear horizontal distance;
y = depth of flow referred to the average bottom elevation at a cross section;
z = elevation referred to an arbitrary horizontal datum;
α = kinetic energy correction factor;
γ = specific weight of the fluid;
ρ = density of the fluid; and
ϕ = acute angle between the plane of the constriction and a line normal to the thread of the stream.

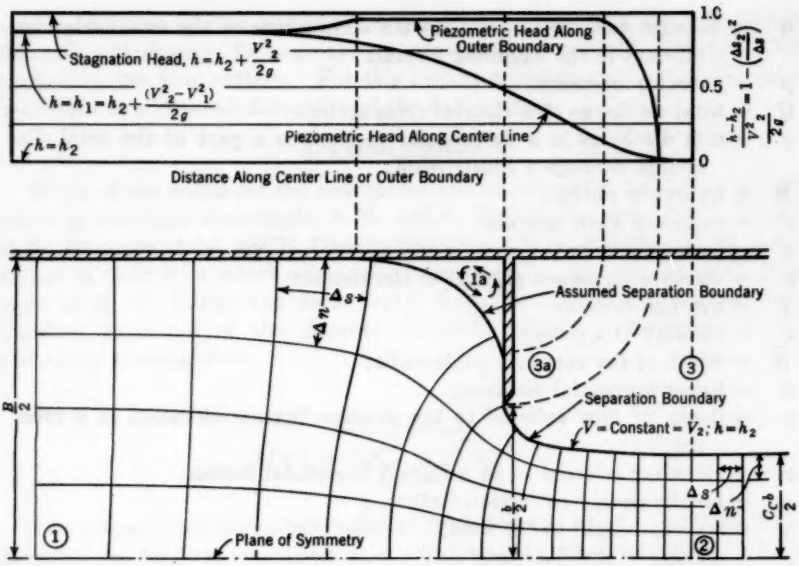

FIG. 1.—SIMPLE WIDTH CONSTRICTION IN A HORIZONTAL RECTANGULAR CHANNEL

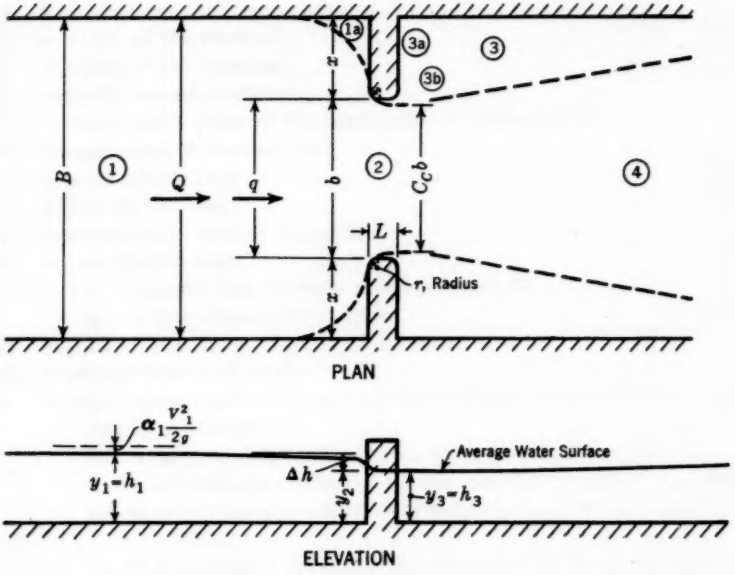

FIG. 2.—TWO-DIMENSIONAL CONSTRICTION WITH $\frac{b}{B} = 0.5$

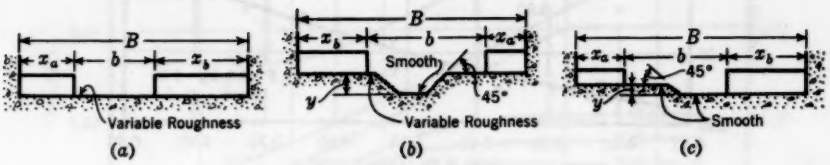

FIG. 3.—VARIATIONS IN SHAPE OF CHANNELS

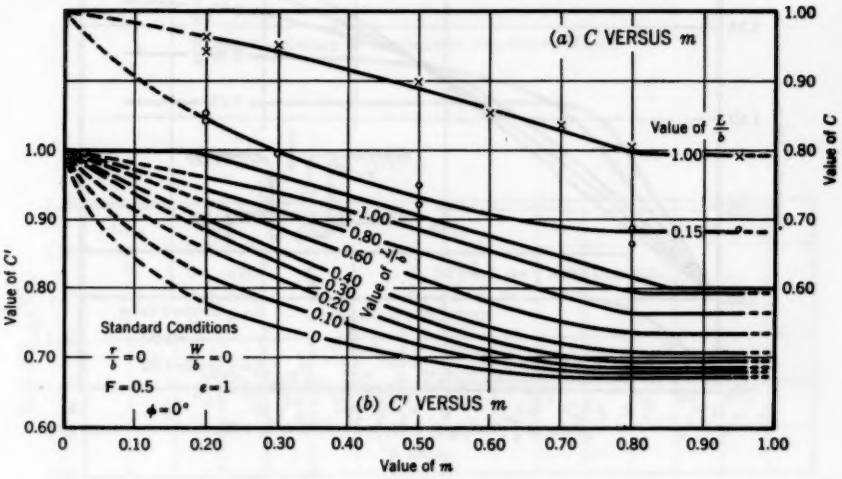

FIG. 4.—VARIATION OF DISCHARGE COEFFICIENT C AND C' WITH m AND L/b

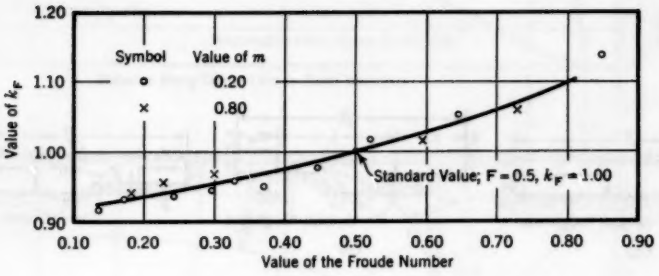

FIG. 5.—EFFECT ON DISCHARGE COEFFICIENT OF THE FROUDE NUMBER

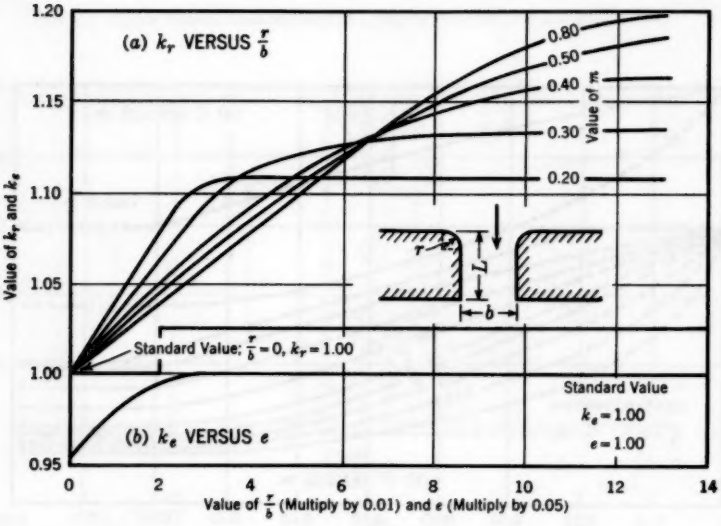

FIG. 6.—EFFECT OF CORNER ROUNDING AND ECCENTRICITY

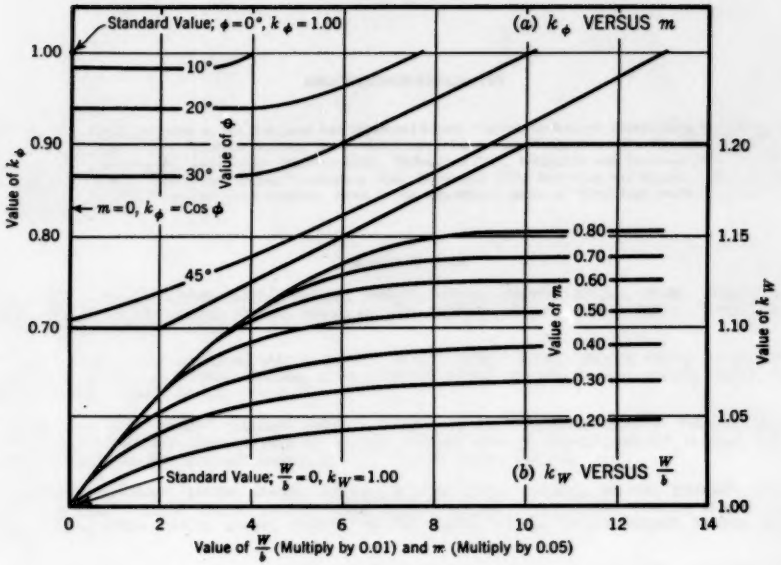

FIG. 7.—EFFECT OF ANGULARITY AND GUIDE WALLS

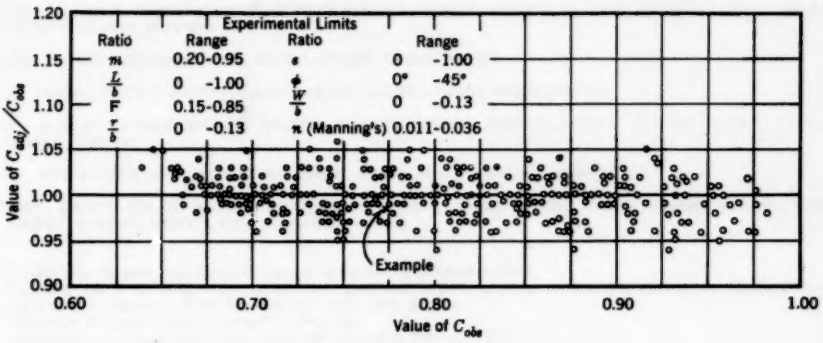

FIG. 8.—COMPARISON OF ADJUSTED AND OBSERVED VALUES OF C

PROCEEDINGS-SEPARATES

The technical papers published in the past year are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Separate Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

VOLUME 79 (1953)

AUGUST: 230(HY), 231(SA), 232(SA), 233(AT), 234(HW), 235(HW), 237(AT), 238(WW), 239(SA), 240(IR), 241(AT), 242(IR), 243(ST), 244(ST), 245(ST), 246(ST), 247(SA), 248(SA), 249(ST), 250(EM)[a], 251(ST), 252(SA), 253(AT), 254(HY), 255(AT), 256(ST), 257(SA), 258(EM), 259(WW).

SEPTEMBER: 260(AT), 261(EM), 262(SM), 263(ST), 264(WW), 265(ST), 266(ST), 267(SA), 268(CO), 269(CO), 270(CO), 271(SU), 272(SA), 273(PO), 274(HY), 275(WW), 276(HW), 277(SU), 278(SU), 279(SA), 280(IR), 281(EM), 282(SU), 283(SA), 284(SU), 285(CP), 286(EM), 287(EM), 288(SA), 289(CO).

OCTOBER:[b] 290(all Divs), 291(ST)[a], 292(EM)[a], 293(ST)[a], 294(PO)[a], 295(HY)[a], 296(EM)[a], 297(HY)[a], 298(ST)[a], 299(EM)[a], 300(EM)[a], 301(SA)[a], 302(SA)[a], 303(SA)[a], 304(CO)[a], 305(SU)[a], 306(ST)[a], 307(SA)[a], 308(PO)[a], 309(SA)[a], 310(SA)[a], 311(SM)[a], 312(SA)[a], 313(ST)[a], 314(SA)[a], 315(SM)[a], 316(AT), 317(AT), 318(WW), 319(IR), 320(HW).

NOVEMBER: 321(ST), 322(ST), 323(SM), 324(SM), 325(SM), 326(SM), 327(SM), 328(SM), 329(HW), 330(EM)[a], 331(EM)[a], 332(EM)[a], 333(EM)[c], 334(EM), 335(SA), 336(SA), 337(SA), 338(SA), 339(SA), 340(SA), 341(SA), 342(CO), 343(ST), 344(ST), 345(ST), 346(IR), 347(IR), 348(CO), 349(ST), 350(HW), 351(HW), 352(SA), 353(SU), 354(HY), 355(PO), 356(CO), 357(HW), 358(HY).

DECEMBER: 359(AT), 360(SM), 361(HY), 362(HY), 363(SM), 364(HY), 365(HY), 366(HY), 367(SU)[c], 368(WW)[c], 369(IR), 370(AT)[c], 371(SM)[c], 372(CO)[c], 373(ST)[c], 374(EM)[c], 375(EM), 376(EM), 377(SA)[c], 378(PO)[c].

VOLUME 80 (1954)

JANUARY: 379(SM)[c], 380(HY), 381(HY), 382(HY), 383(HY), 384(HY)[c], 385(SM), 386(SM), 387(EM), 388(SA), 389(SU)[c], 390(HY), 391(IR)[c], 392(SA), 393(SU), 394(AT), 395(SA)[c], 396(EM)[c], 397(ST)[c].

FEBRUARY: 398(IR)[d], 399(SA)[d], 400(CO)[d], 401(SM)[c], 402(AT)[d], 403(AT)[d], 404(IR)[d], 405(PO)[d], 406(AT)[d], 407(SU)[d], 408(SU)[d], 409(WW)[d], 410(AT)[d], 411(SA)[d], 412(PO)[d], 413(HY)[d].

MARCH: 414(WW)[d], 415(SU)[d], 416(SM)[d], 417(SM)[d], 418(AT)[d], 419(SA)[d], 420(SA)[d], 421(AT)[d], 422(SA)[d], 423(CP)[d], 424(AT)[d], 425(SM)[d], 426(IR)[d], 427(WW)[d].

APRIL: 428(HY)[c], 429(EM)[c], 430(ST), 431(HY), 432(HY), 433(HY), 434(ST).

MAY: 435(SM), 436(CP)[c], 437(HY)[c], 438(HY), 439(HY), 440(ST), 441(ST), 442(SA), 443(SA).

JUNE: 444(SM)[e], 445(SM)[e], 446(ST)[e], 447(ST)[e], 448(ST)[e], 449(ST)[e], 450(ST)[e], 451(ST)[e], 452(SA)[e], 453(SA)[e], 454(SA)[e], 455(SA)[e], 456(SM)[e].

JULY: 457(AT), 458(AT), 459(AT)[c], 460(IR), 461(IR), 462(IR), 463(IR)[c], 464(PO), 465(PO)[c].

AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)[c], 479(HY)[c], 480(ST)[c], 481(SA)[c], 482(HY), 483(HY).

---

a. Presented at the New York (N.Y.) Convention of the Society in October, 1953.
b. Beginning with "Proceedings-Separate No. 290," published in October, 1953, an automatic distribution of papers was inaugurated, as outlined in "Civil Engineering," June, 1953, page 66.
c. Discussion of several papers, grouped by Divisions.
d. Presented at the Atlanta (Ga.) Convention of the Society in February, 1954.
e. Presented at the Atlantic City (N.J.) Convention in June, 1954.

AMERICAN SOCIETY OF CIVIL ENGINEERS

OFFICERS FOR 1954

PRESIDENT

DANIEL VOIERS TERRELL

VICE-PRESIDENTS

Term expires October, 1954:
EDMUND FRIEDMAN
G. BROOKS EARNEST

Term expires October, 1955:
ENOCH R. NEEDLES
MASON G. LOCKWOOD

DIRECTORS

Term expires October, 1954:
WALTER D. BINGER
FRANK A. MARSTON
GEORGE W. McALPIN
JAMES A. HIGGS
I. C. STEELE
WARREN W. PARKS

Term expires October, 1955:
CHARLES B. MOLINEAUX
MERCEL J. SHELTON
A. A. K. BOOTH
CARL G. PAULSEN
LLOYD D. KNAPP
GLENN W. HOLCOMB
FRANCIS M. DAWSON

Term expires October, 1956:
WILLIAM S. LaLONDE, JR.
OLIVER W. HARTWELL
THOMAS C. SHEDD
SAMUEL B. MORRIS
ERNEST W. CARLTON
RAYMOND F. DAWSON

PAST-PRESIDENTS

Members of the Board

CARLTON S. PROCTOR
WALTER L. HUBER

EXECUTIVE SECRETARY
WILLIAM N. CAREY

TREASURER
CHARLES E. TROUT

ASSISTANT SECRETARY
E. L. CHANDLER

ASSISTANT TREASURER
GEORGE W. BURPEE

PROCEEDINGS OF THE SOCIETY

HAROLD T. LARSEN
Manager of Technical Publications

DEFOREST A. MATTESON, JR.
Editor of Technical Publications

PAUL A. PARISI
Assoc. Editor of Technical Publications

COMMITTEE ON PUBLICATIONS

FRANK A. MARSTON, *Chairman*

I. C. STEELE
GLENN W. HOLCOMB
ERNEST W. CARLTON
OLIVER W. HARTWELL
SAMUEL B. MORRIS